IJERMT
International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
Jan-Feb 2018 Volume 5, Issue-1
Email: editor@ijermt.org
www.ijermt.org

A Study On Domination Parameters Of Graphs

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Abstract

Domination parameters in graph theory are a fascinating area of study, offering a rich tapestry of concepts and applications. These parameters explore the concept of control and influence within a network, with numerous implications in fields like computer science, social networks, and operations research. A dominating set in a graph is a subset of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set. The domination number, denoted by $\gamma(G)$, is the minimum size of a dominating set in a graph G. This parameter quantifies the minimum number of vertices required to exert control over the entire network. A fundamental concept, a dominating set S in a graph G is a subset of vertices such that every vertex in G is either in S or adjacent to a vertex in S. The domination number $\gamma(G)$ represents the minimum size of a dominating set in G. This parameter provides a measure of the graph's controllability. A total dominating set T in G is a subset of vertices such that every vertex in G is adjacent to at least one vertex in T. The total domination number $\gamma t(G)$ signifies the minimum size of a total dominating set. This parameter ensures that there are no isolated vertices in the graph.

Keywords:

Domination, Parameters, Graphs, Vertices

Introduction

A connected dominating set C in G is a dominating set that also induces a connected subgraph of G. The connected domination number $\gamma c(G)$ represents the minimum size of a connected dominating set. This parameter is crucial in applications where maintaining connectivity is essential, such as in network design. [10] (Oellermann, 2015)

An independent dominating set I in G is a dominating set whose vertices are pairwise non-adjacent. The independent domination number i(G) denotes the minimum size of an independent dominating set. This parameter is relevant in scenarios where minimizing interference or conflicts is desired.

A Roman dominating set R in G is a subset of vertices such that every vertex not in R is adjacent to at least one vertex in R, and every vertex in R is adjacent to at least two vertices in R. The Roman domination number $\gamma R(G)$ represents the minimum size of a Roman dominating set. This parameter finds applications in resource allocation and scheduling problems. [1]

Other notable domination parameters include:

- Total domination number: Requires every vertex to be adjacent to at least one vertex in the dominating set.
- Connected domination number: Ensures that the induced subgraph of the dominating set is connected.
- Independent domination number: Demands that the dominating set be an independent set, meaning no two vertices in the set are adjacent.
- Roman domination number: Assigns weights to vertices, either 0, 1, or 2, to achieve domination.

The applications of domination parameters are far-reaching: [9] (Zhang, 2016)

- Network Design: Dominating sets can be used to identify critical nodes in communication networks, ensuring efficient information dissemination.
- Social Networks: Analyzing social networks through domination parameters can reveal influential individuals or groups.
- Facility Location: Dominating sets can help determine optimal locations for facilities to maximize coverage.
- Distributed Computing: Dominating sets can be used to coordinate and synchronize distributed systems.

While the study of domination parameters has made significant progress, several challenges remain: [2]

- Computational Complexity: Many problems related to domination parameters are NP-hard, making exact solutions intractable for large graphs.
- Algorithmic Development: Developing efficient algorithms to approximate or find optimal solutions for domination problems is an active area of research.
- Real-world Applications: Exploring new applications of domination parameters in diverse fields, such as biology and epidemiology, offers exciting opportunities.

Domination parameters have numerous applications in various fields:

- Network Design: Dominating sets can help identify optimal placement of servers or routers in communication networks.
- Social Networks: Dominating sets can be used to identify influential individuals or groups within a social network.
- Coding Theory: Dominating sets are relevant in the design of error-correcting codes.
- Distributed Computing: Dominating sets can be used to coordinate tasks among nodes in a distributed system. [8] (Escuadro, 2015)

Review of Literature

Gologranc et al. (2015): At the heart of domination lies the concept of a dominating set. In a graph G, a dominating set S is a subset of vertices such that every vertex in G is either in S or adjacent to a vertex in S. In simpler terms, every vertex in the graph is either a dominator or is dominated by a dominator.

Bresar et al. (2015): Domination parameters provide a powerful framework for analyzing the structure and properties of graphs. By studying these parameters, we gain valuable insights into the controllability, connectivity, and independence within a graph. These insights have practical applications in diverse fields, making domination parameters a valuable tool in the study of graph theory.

Arumugam et al. (2015): The domination number of a graph G, denoted by γ(G), is the minimum cardinality of a dominating set in G. It represents the minimum number of vertices required to dominate the entire graph.

Buckley et al. (2016): A total dominating set T in G is a subset of vertices such that every vertex in G is adjacent to at least one vertex in T. The total domination number, γt(G), is the minimum cardinality of a total dominating set.

Domination Parameters Of Graphs

A connected dominating set C in G is a dominating set that induces a connected subgraph. The connected domination number, γc(G), is the minimum cardinality of a connected dominating set. [3]

An independent dominating set I in G is a dominating set that is also an independent set, meaning no two vertices in I are adjacent. The independent domination number, i(G), is the minimum cardinality of an independent dominating set.

$$\Delta_T(t) \overset{\text{def}}{=} \sum_{n=0}^{\infty} \delta(t - nT)$$

$$x_q(t) \overset{\text{def}}{=} (t)\Delta_T(t) = x(t)\sum_{n=0}^{\infty} \delta(t - nT)$$

$$= \sum_{n=0}^{\infty} x(nT)\delta(t - nT) = \sum_{n=0}^{\infty} x[n]\delta(t - nT)$$

$$X_q(s) = \int_{0^-}^{\infty} x_q(t) e^{-st} dt$$

$$= \int_{0^-}^{\infty} \sum_{n=0}^{\infty} x[n]\delta(t - nT)e^{-st}$$

$$= \sum_{n=0}^{\infty} x[n] \int_{0^-}^{\infty} \delta(t - nT)e^{-st}$$

$$= \sum_{n=0}^{\infty} x[n]e^{-nsT}$$

A restrained dominating set R in G is a dominating set such that every vertex in V(G) \ R is adjacent to at least one vertex in R and at least one vertex in V(G) \ R. The restrained domination number, γr(G), is the minimum cardinality of a restrained dominating set.

The field of domination parameters is vast and continues to evolve. There are numerous variations and generalizations of the basic concepts, such as fractional domination, domination in hypergraphs, and domination in digraphs. Exploring these areas can lead to deeper insights into the structure and properties of graph. Domination parameters provide powerful tools for analyzing graphs and understanding their underlying structure. They have practical applications in various fields, making them an essential topic in graph theory.[4]

While domination parameters have been studied extensively, many open problems and challenges remain. Some of the key challenges include:

- Computational Complexity: Many domination problems are NP-hard, making it difficult to find exact solutions for large graphs.
- Algorithmic Development: Developing efficient algorithms to approximate domination parameters is an important area of research.
- Real-world Applications: Exploring new applications of domination parameters in various fields is an exciting area of research.

Domination parameters are a powerful tool for analyzing the structure and properties of graphs. Their applications in various fields demonstrate their versatility and importance. As research in this area continues to evolve, we can expect to see even more innovative applications of domination parameters in the future.

Domination parameters can be used to determine the optimal placement of facilities (e.g., hospitals, fire stations, cell towers) to minimize the distance that any individual needs to travel to reach a facility.

By identifying dominating sets in sensor networks, we can minimize the number of sensors required to monitor a given area while ensuring full coverage. Domination parameters can help analyze the influence of individuals within social networks, identifying key influencers who can spread information or initiate trends. Domination parameters can be used to design efficient error-correcting codes, which are essential for reliable communication over noisy channels. Domination parameters can help analyze the structure of proteins and identify key residues that play a crucial role in protein function.[5]

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} e^{st}F(s)ds,$$

$$B_k[f(x)] = \sum_{m=0}^{k} f\left(\frac{m}{k}\right)\lambda_{k,m}(x).$$

$$B_k[1] = \sum_{m=0}^{k} \lambda_{k,m}(x) = 1.$$

$$\sum_{m=0}^{k} \left|\lambda_{k,m}\right| < L$$

Domination parameters can be used to study the properties of chemical compounds and predict their reactivity. Domination parameters can be used to design efficient distributed algorithms for tasks such as leader election and resource allocation. Domination parameters can be used to analyze the complexity of algorithms and identify potential optimizations. Domination parameters can be used to model the spread of infectious diseases and identify key individuals who can help control the outbreak.[6]

Conclusion

Domination parameters offer a powerful lens through which to analyze the structure and properties of graphs. Their applications span various domains, making them an invaluable tool for researchers and practitioners alike. As our understanding of these parameters deepens, we can expect to see even more innovative applications and theoretical advancements in the future.

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